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FINITE-AMPLITUDE STABILITY ANALYSIS OF LIQUID FILMS DOWN A VERTICAL WALL WITH AND WITHOUT INTERFACIAL PHASE CHANGE

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Abstract—The generalized kinematic equation for film thickness, taking into account the effect of phase change at the interface, is used to investigate the nonlinear stability of film flow down a vertical wall. The analysis shows that supercritical stability and subcritical instability are both possible for the film flow system. Applications of the result to isothermal, condensate and evaporate film flow show that mass transfer into (away from) the liquid phase will stabilize (destabilize) the film flow. Finally, we find that supercritical filtered waves are always linearly stable with regard to side-band disturbance.

1. INTRODUCTION

The problem of stability of isothermal film flow, without taking into account surface tension, was first considered by Yih (1954). Subsequently, the analysis of this problem was refined by many authors, including Benjamin (1957), Yih (1963), Anshus (1965), Gjevik (1970), Lin (1971, 1974), Nakaya (1975). It was found that the film flow is linearly unstable for all finite Reynolds number. Also, Anshus (1965) reported that nonlinear instability occurred in the region near the upper branch of the neutral stability curve, however, Lin (1974) reported such instability as occurring near the lower branch of the neutral stability curve in the α–Re plane.

The original theory of gravity-induced laminar film condensation flow was developed by Nusselt (1916), but the stability analysis of film flow with phase change was never investigated until the 1970s. Bankoff (1971), Marshall & Lee (1973) and Lin (1975) successively presented stability analyses of condensate film flow. They showed that the critical Reynolds number is so small in all practical condensation processes that the film can be assumed to be unstable. However, in these studies, the mass transfer through the phase change at the interface was not considered. Later, Ünsal & Thomas (1978), Spindler (1982) and Kocamustafaogullari (1985) presented this problem in a more detailed form. Their results point out that condensation is a stabilizing effect but evaporation is a destabilizing effect. Ünsal & Thomas (1980) also presented the nonlinear stability analysis of condensate film flow, but there were some errors in their report and only disturbance of the same mode was considered.

In this paper, we present the analysis of finite-amplitude side-band stability of film flow down a vertical wall with phase change at the interface. The method of multiple scales is applied to solve the nonlinear generalized kinematic equation order by order and leads to a secular equation of Ginzburg–Landau type. Then the finite-amplitude and linear stability characteristics of a supercritical wave due to side-band disturbance are investigated. It will be seen that the results in the present study encompass the qualitative features of the different results of the above-mentioned studies.

2. GENERALIZED KINEMATIC EQUATION

The following derivation of governing equations and boundary conditions closely follows the formulation of Ünsal & Thomas (1980), but with a different process to formulate a generalized kinematic equation for the free surface. Consider a layer of an incompressible viscous fluid with phase change at the interface flowing down a vertical plane, as shown in figure 1. The governing equations are two-dimensional mass, momentum and energy balance equations in the liquid phase, the boundary conditions at the wall are the nonslip condition of velocity and a constant wall temperature. The boundary conditions at the liquid–vapor interface are the balance of normal and tangential stresses, the relation of interfacial energy balances and the equality of liquid and saturated vapor temperatures. We assume all physical properties are constant and obtain the equations and boundary conditions as follows:

$$\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}=0, \qquad [1]$$

$$\frac{\partial u^*}{\partial t^*}+u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=-\frac{1}{\rho}\frac{\partial P^*}{\partial x^*}+\nu\left(\frac{\partial^2 u^*}{\partial x^{*2}}+\frac{\partial^2 u^*}{\partial y^{*2}}\right)+g, \qquad [2]$$

$$\frac{\partial v^*}{\partial t^*}+u^*\frac{\partial v^*}{\partial x^*}+v^*\frac{\partial v^*}{\partial y^*}=-\frac{1}{\rho}\frac{\partial P^*}{\partial y^*}+\nu\left(\frac{\partial^2 v^*}{\partial x^{*2}}+\frac{\partial^2 v^*}{\partial y^{*2}}\right), \qquad [3]$$

$$\frac{\partial T}{\partial t^*}+u^*\frac{\partial T}{\partial x^*}+v^*\frac{\partial T}{\partial y^*}=\frac{K}{\rho C_p}\left(\frac{\partial^2 T}{\partial x^{*2}}+\frac{\partial^2 T}{\partial y^{*2}}\right), \qquad [4]$$

$$u^*=0,\quad v^*=0,\quad T=T_w \quad \text{at} \quad y^*=0; \qquad [5]$$

$$P^*+2\rho\nu\frac{\partial u^*}{\partial x^*}\left[1+\left(\frac{\partial h^*}{\partial x^*}\right)^2\right]\left[1-\left(\frac{\partial h^*}{\partial x^*}\right)^2\right]^{-1}+\sigma\frac{\partial^2 h^*}{\partial x^{*2}}\left[1+\left(\frac{\partial h^*}{\partial x^*}\right)^2\right]^{-3/2}$$
$$+K^2 h_{fG}^{-2}\rho^{-1}\gamma^{-1}(\gamma-1)\left(\frac{\partial T}{\partial y^*}-\frac{\partial h^*}{\partial x^*}\frac{\partial T}{\partial x^*}\right)^2\left[1+\left(\frac{\partial h^*}{\partial x^*}\right)^2\right]^{-1}=P_G^*, \qquad [6]$$

$$\frac{\partial u^*}{\partial y^*}+\frac{\partial v^*}{\partial x^*}+4\frac{\partial h^*}{\partial x^*}\frac{\partial u^*}{\partial x^*}\left[\left(\frac{\partial h^*}{\partial x^*}\right)^2-1\right]^{-1}=0, \qquad [7]$$

$$K\left(\frac{\partial T}{\partial y^*}-\frac{\partial h^*}{\partial x^*}\frac{\partial T}{\partial x^*}\right)-\rho h_{fG}\left(\frac{\partial h^*}{\partial t^*}+u^*\frac{\partial h^*}{\partial x^*}-v^*\right)=0, \qquad [8]$$

$$T=T_s \quad \text{at} \quad y^*=h^*. \qquad [9]$$

Where

u^*, v^* = velocity in the x^*- and y^*-directions, respectively,
t^* = time,
P^* = pressure,
g = gravitational acceleration,
ν = kinematic viscosity,
ρ = density,
K = thermal conductivity,
T = temperature,
T_w = wall temperature,
T_s = saturated vapor temperature,
C_p = specific heat,
h_{fG} = latent heat of phase change,
P_G^* = vapor pressure,
σ = surface tension,
γ = ratio of vapor density to liquid density

and

h^* = height of film thickness.

We introduce the stream function ψ^* and the following dimensionless quantities:

$$\left.\begin{aligned}
\left(\frac{\partial\psi^*}{\partial y^*}, -\frac{\partial\psi^*}{\partial x^*}\right) &= (u^*, v^*),\\
\psi &= \frac{\psi^*}{U_0^* h_0^*},\\
U_0^* &= \frac{g(1-\gamma)h_0^{*2}}{2\nu},\\
\alpha &= \frac{2\pi h_0^*}{\lambda},\\
P &= \frac{(P^* - P_G^*)}{\rho U_0^{*2}},\\
h &= \frac{h^*}{h_0^*},\\
\theta &= \frac{T - T_w}{T_s - T_w},\\
(x, y, t) &= \left(\frac{\alpha x^*}{h_0^*}, \frac{y^*}{h_0^*}, \frac{\alpha t^* U_0^*}{h_0^*}\right),\\
\mathrm{Re} &= \frac{h_0^* U_0^*}{\nu},\\
W &= \left(\frac{2\sigma^3}{\rho^3 \nu^4 g(1-\gamma)}\right)^{\frac{1}{3}},\\
\xi &= \frac{C_p \Delta T}{h_{fG}},\\
\mathrm{Nd} &= \frac{\xi^2}{\gamma \mathrm{Pr}},\\
\mathrm{Pr} &= \frac{\rho \nu C_p}{K},\\
\mathrm{Pe} &= \mathrm{Pr}\cdot\mathrm{Re},
\end{aligned}\right\} \qquad [10]$$

where the value of A, B and their derivations are evaluated at $h = 1$.

For the linear stability analysis, we neglect the nonlinear part of [25] and obtain the linearized equation

$$\psi_{yyy} = -2 + \alpha\,\mathrm{Re}(P_x + \psi_{yt} + \psi_y\psi_{xy} - \psi_x\psi_{yy}) - \alpha^2\psi_{xxy}, \qquad [11]$$

$$P_y = -\alpha\,\mathrm{Re}^{-1}\psi_{xyy} + \alpha^2(\psi_y\psi_{xx} - \psi_x\psi_{xy} + \psi_{xt}) - \alpha^2\mathrm{Re}^{-1}\psi_{xxx}, \qquad [12]$$

$$\psi_{yy} = \alpha\,\mathrm{Pe}(\psi_y\theta_x - \psi_x\theta_y + \theta_t) - \alpha^2\theta_{xx}, \qquad [13]$$

$$\psi = \psi_x = \psi_y = 0, \quad \theta = 0 \quad \text{at} \quad y = h; \qquad [14]$$

$$P + 2\alpha\,\mathrm{Re}^{-1}\psi_{xy}(1 + \alpha^2 h_x^2)(1 - \alpha^2 h_x^2)^{-1} + \alpha^2 W \mathrm{Re}^{-\frac{5}{3}} h_{xx}$$
$$\times (1 + \alpha^2 h_x^2)^{-3/2} + (\gamma - 1)\mathrm{Nd}\,\mathrm{Re}^{-2}(\theta_y - \alpha^2 h_x\theta_x)^2(1 + \alpha^2 h_x^2)^{-1} = 0, \qquad [15]$$

$$\psi_{yy} = \alpha^2\psi_{xx} + 4\alpha^2\psi_{xy}h_x(1 - \alpha^2 h_x^2)^{-1}, \qquad [16]$$

$$\xi(\theta_y - \alpha^2 h_x\theta_x) - \alpha\,\mathrm{Pe}(h_t + \psi_y h_x + \psi_x) = 0, \qquad [17]$$

$$\theta = 1 \quad \text{at} \quad y = h. \qquad [18]$$

It is noted that [17], derived from the energy balance, will be used to determine the time evolution of the film thickness, and we call it the generalized kinematic condition. If $\xi > 0$, then [17] is used

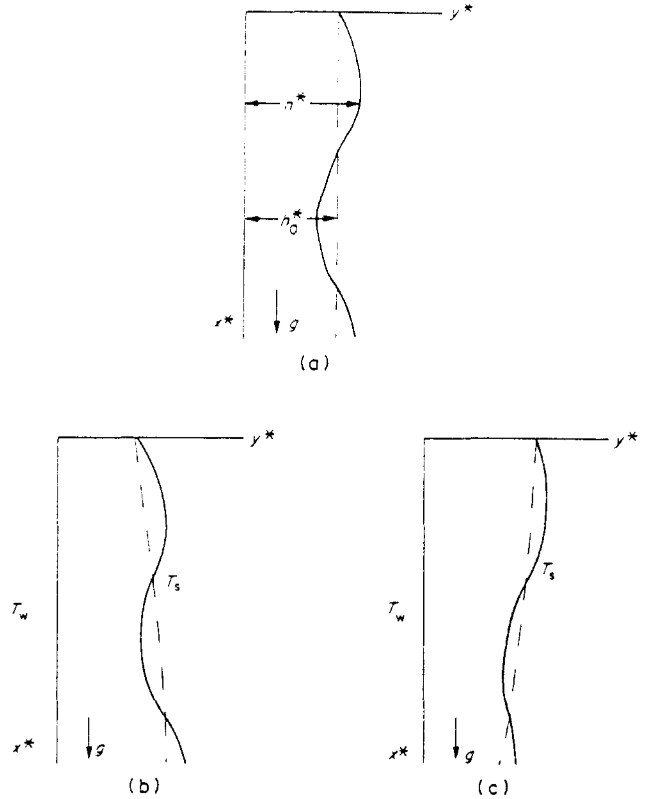

Figure 1. The flow system: (a) isothermal film flow; (b) condensate film flow; (c) evaporate film flow.

for the case of condensate film flow; if $\xi > 0$, [17] is used for evaporative film flow; and if $\xi = 0$, [17] reduces to the usual form of the kinematic condition for isothermal film flow.

Since the long-wavelength (small wavenumber α) modes are the most unstable ones for film flow, we expand ψ, P and θ in the following form:

$$\left.\begin{aligned} \psi &= \psi_0 + \alpha\psi_1 + \cdots, \\ P &= P_0 + \alpha P_1 + \cdots, \\ \theta &= \theta_0 + \alpha\theta_1 + \cdots. \end{aligned}\right\} \qquad [19]$$

The above expression is then substituted into the system [11–18], which is then solved order by order. The zeroth- and first-order solutions are then substituted into [17] and h_t, which appeared in the first-order solution of [17], eliminated. This yields the following nonlinear generalized kinematic equation, which is simpler to handle:

$$h_t + A(h) + B(h)h_x + C(h)h_{xx} + D(h)h_{xxxx} + E(h)h_x^2 + F(h)h_x h_{xxx} + O(\alpha^2) = 0, \qquad [20]$$

where

$$A(h) = -\frac{\xi}{\alpha\,\mathrm{Pe}}\left(1 - \frac{\xi}{3}\right)\frac{1}{h},$$

$$B(h) = \left(2 - \frac{7}{60}\xi - \frac{5\xi}{4\,\mathrm{Pr}}\right)h^2,$$

$$C(h) = \frac{8}{15}\alpha\,\mathrm{Re}\,h^6,$$

$$D(h) = \frac{\alpha^2}{3}W\,\mathrm{Re}^{-\frac{1}{2}}h^3,$$

$$E(h) = -\frac{\alpha\xi}{\mathrm{Pe}}\left(1 - \frac{\xi}{3}\right)\frac{1}{h} + \frac{16}{5}\alpha\,\mathrm{Re}\,h^5$$

and

$$F(h) = \alpha^3 W\,\mathrm{Re}^{-2/3} h^2.$$

3. STABILITY ANALYSIS

If the Nusselt assumption is adopted for the base flow, then [22], for the unperturbed state (only ψ_0 and θ_0 are considered here), is

$$2h^2 h_x - \frac{\xi}{\alpha\,\mathrm{Pr}\,\mathrm{Re}}\frac{1}{h} = 0. \qquad [21]$$

From which it is easy to find

$$\alpha h_x = \left(\frac{\xi}{2\,\mathrm{Pr}\,\mathrm{Re}}\right)h^{-3}. \qquad [22]$$

If $|\alpha h_x| \ll 1$, i.e. the variation of film thickness for the base flow is very small, then it is reasonable to assume that the local nondimensional thickness = 1. Thus, the nondimensional film thickness for the perturbed state may be expanded in the following form:

$$h = 1 + \eta \qquad [23]$$

where η is the perturbation of the thickness. From [22], it is clear that when

$$\mathrm{Re} \gg \frac{|\xi|}{2\,\mathrm{Pr}}, \qquad [24]$$

the above expansion is valid. Since ξ is usually very small the expansion of [23] may be used for a wide range of applications.

Substituting [23] into [20], keeping terms up to $O(\eta^3)$, leads to the evolution of η:

$$\begin{aligned}\eta_t &+ A'\eta + B\eta_x + C\eta_{xx} + D\eta_{xxxx}\\ &= -\left[\frac{A''}{2}\eta^2 + \frac{A'''}{6}\eta^3 + \left(B'\eta + \frac{B''}{2}\eta^2\right)\eta_x + \left(C''\eta + \frac{C''}{2}\eta^2\right)\eta_{xx}\right.\\ &\quad \left. + \left(D'\eta + \frac{D''}{2}\eta^2\right)\eta_{xxxx} + (E + E'\eta)\eta_x^2 + (F + F'\eta)\eta_x\eta_{xx}\right] + O(\eta^4),\end{aligned} \qquad [25]$$

where the value of A, B and their derivations are evaluated at $h = 1$.

For the linear stability analysis, we neglect the nonlinear part of [25] and obtain the linearized equation

$$\frac{\partial\eta}{\partial t} + A'\eta + B\frac{\partial\eta}{\partial x} + C\frac{\partial^2\eta}{\partial x^2} + D\frac{\partial^4\eta}{\partial x^4} = 0. \qquad [26]$$

Assuming the normal mode solution to be

$$\eta = \Gamma\exp[i(x - dt)] + \bar{\Gamma}\exp[-i(x - dt)], \qquad [27]$$

the complex wave celerity corresponding to the linear stability problem is given by

$$\begin{aligned}d &= d_r + id_i = B + i(C - D - A')\\ &= \left(2 - \frac{7\xi}{60} - \frac{5}{4}\frac{\xi}{\mathrm{Pr}}\right) + i\left[\frac{8}{15}\alpha\,\mathrm{Re} - \frac{\alpha^3}{3}W\,\mathrm{Re}^{-2/3} - \frac{\xi}{\alpha\,\mathrm{Re}}\left(1 - \frac{\xi}{3}\right)\right].\end{aligned} \qquad [28]$$

It is noted that, from the expression for the wave speed d_r, long waves in a liquid film travel at approximately twice the speed of the unperturbed surface. Also $d_i = 0$ gives the neutral stability curve,

$$\frac{8}{15}\alpha^2\,\mathrm{Re}-\frac{\alpha^4}{3}W\,\mathrm{Re}^{-2/3}-\frac{\xi}{\mathrm{Re}}\left(1-\frac{\xi}{3}\right)=0. \qquad [29]$$

For the nonlinear stability analysis, we use the method of multiple scales, according to

$$\frac{\partial}{\partial t}\rightarrow\frac{\partial}{\partial t}+\epsilon\frac{\partial}{\partial t_1}+\epsilon^2\frac{\partial}{\partial t_2},\quad \frac{\partial}{\partial x}\rightarrow\frac{\partial}{\partial x}+\epsilon\frac{\partial}{\partial x_1},\quad \eta(\alpha,x,x_1,t,t_1,t_2)=\epsilon\eta_1+\epsilon^2\eta_2+\epsilon^3\eta_3, \qquad [30]$$

where ϵ is a small parameter, then [25] becomes

$$(L_0+\epsilon L_1+\epsilon^2L_2)(\epsilon\eta_1+\epsilon^2\eta_2+\epsilon^3\eta_3)=-\epsilon^2N_2-\epsilon^3N_3, \qquad [31]$$

where

$$L_0=\frac{\partial}{\partial t}+A'+B\frac{\partial}{\partial x}+C\frac{\partial^2}{\partial x^2}+D\frac{\partial^4}{\partial x^4},$$

$$L_1=\frac{\partial}{\partial t_1}+B\frac{\partial}{\partial x_1}+2C\frac{\partial}{\partial x}\frac{\partial}{\partial x_1}+4D\frac{\partial^3}{\partial x^3}\frac{\partial}{\partial x_1},$$

$$L_2=\frac{\partial}{\partial t_2}+C\frac{\partial^2}{\partial x_1^2}+6D\frac{\partial}{\partial x^2}\frac{\partial^2}{\partial x_1^2},$$

$$N_2=\frac{A''}{2}\eta_1^2+B'\eta_1\eta_{1x}+C'\eta_1\eta_{1xxx}+D'\eta_1\eta_{1xxxx}+E\eta_{1x}^2+F\eta_{1x}\eta_{1xxx}$$

and

$$N_3=A''\eta_1\eta_2+\frac{A''}{6}\eta_1^3+B'(\eta_1\eta_{2x}+\eta_1\eta_{1x_1}+\eta_{1x}\eta_2)+\frac{B''}{2}\eta_1^2\eta_{1x}+C'(\eta_{1xx}\eta_2+\eta_1\eta_{2xx}+2\eta_1\eta_{1xx_1})$$
$$+\frac{C''}{2}\eta_1^2\eta_{1xx}+D'(\eta_{1xxxx}\eta_2+\eta_1\eta_{2xxxx_1}+4\eta_1\eta_{1xxxx_1})+\frac{D''}{2}\eta_1^2\eta_{1xxxx}+E(2\eta_{1x}\eta_{2x}$$
$$+2\eta_{1x_1}\eta_{1x})+E'\eta_1\eta_{1x}^2+F(\eta_{1xxx}\eta_{2x}+\eta_{1xxx}\eta_{1x_1}+\eta_{1x}\eta_{1xxx_1}+\eta_{1x}\eta_{2xxx})+F'\eta_1\eta_{1x}\eta_{1xxx}.$$

Equation [31] is solved order by order; the $O(\epsilon)$ equation is $L_0\eta_1=0$, the solution of which is in the form

$$\eta_1=\Gamma(x_1,t_1,t_2)\exp[i(x-d_r t)]+\text{C.C.}, \qquad [32]$$

then the solution of η_2 and the secular condition for $O(\epsilon^3)$ are

$$\eta_2=C_1\Gamma^2\exp[2i(x-d_r t)]+\text{C.C.} \qquad [33]$$

and

$$\frac{\partial\Gamma}{\partial t_2}+D_1\frac{\partial^2\Gamma}{\partial x_1^2}-\epsilon^{-2}d_i\Gamma+(E_1+iF_1)\Gamma^2\bar{\Gamma}=0, \qquad [34]$$

respectively, where

$$C_1=C_{ir}+iC_{1i}=(16D-4C-A)^{-1}[(A+12C-6D)-2iB],$$

$$D_1=C-6D,$$

$$E_1=(-4A-15C+3D)+(4A-6C+21D)C_{1r}-2BC_{1i}$$

and

$$F_1=B+2BC_{1r}+(4A-6C+21D)C_{1i}$$

We shall use [34] to study the nonlinear behavior of film flow. For a filtered wave, there is no spatial modulation and the diffusion term in [34] vanishes. The solution of this equation may be written as

$$\Gamma_\infty=a\exp(-ibt_2). \qquad [35]$$

Substituting [35] into [34], neglecting the second term, we obtain the following results:

$$\epsilon a = \left(\frac{d_i}{E_1}\right)^{1/2} \quad [36a]$$

and

$$\epsilon^2 b = F_1\left(\frac{d_i}{E_1}\right). \quad [36b]$$

From the form of ϵa one knows that in the linear unstable region ($d_i > 0$) the condition for existence of a supercritical wave is $E_1 > 0$ and $2\epsilon a$ is just the final amplitude. On the other hand, in the linear stable region ($d_i < 0$) if $E_1 < 0$, then the film flow has the behavior of subcritical instability and $2\epsilon a$ is the threshold amplitude.

4. SIDE-BAND STABILITY ANALYSIS

It is known from experiments that precisely controlling the wave motion at a given mode is an extremely difficult task. In laboratories, the presence of side-band disturbance is practially unavoidable. Studies on the mechanism of this disturbance have been published by Eckhaus (1965), Stuart & Diprima (1978) and Keffe (1985). In this section we examine whether a filtered finite-amplitude wave of film flow with phase change is stable with respect to side-band disturbance. Let the amplitude Γ be in the following form:

$$\Gamma = \Gamma_\infty + \delta[\Gamma_1(t_2)\exp(iG_1x_1) + \Gamma_2(t_2)\exp(-iG_1x_1)]\exp(-ibt_2), \quad [37]$$

where Γ_∞ represents the equilibrium state and the other term represents the side-band disturbance with $\delta \ll 1$ and $G_1 = O(1)$. Substituting this expression into [34] and collecting the coefficients of $\delta\exp[i(G_1x_1 - bt_2)]$ and $\delta\exp[-i(G_1x_1 + bt_2)]$, the linear coupled equations of Γ_1 and $\bar{\Gamma}_2$ are obtained as follows:

$$\frac{\partial}{\partial t_2}\begin{bmatrix}\Gamma_1\\ \bar{\Gamma}_2\end{bmatrix} + \begin{bmatrix}H_{11} & H_{12}\\ H_{21} & H_{22}\end{bmatrix}\begin{bmatrix}\Gamma_1\\ \bar{\Gamma}_2\end{bmatrix} = 0, \quad [38]$$

where

$$H_{11} = \epsilon^{-2}d_i - D_1G_1 + i\epsilon^{-2}d_i\left(\frac{F_1}{E_1}\right),\quad H_{12} = (E_1 + iF_1)\left(\frac{\epsilon^{-2}d_i}{E_1}\right),\quad H_{21} = (E_1 - iF_1)\left(\frac{\epsilon^{-2}d_i}{E_1}\right)$$

and

$$H_{22} = \epsilon^{-2}d_i - D_1G_1 - i\epsilon^{-2}d_i\left(\frac{F_1}{E_1}\right).$$

Considering only the linear stability of the supercritical stable wave, one can write the solution of [38] as

$$\begin{bmatrix}\Gamma_1\\ \bar{\Gamma}_2\end{bmatrix} = \begin{bmatrix}\Lambda_1\\ \Lambda_2\end{bmatrix}\exp(\Lambda t_2); \quad [39]$$

the eigenvalue Λ in the above solution, after solving the eigenvalue problem of [38], is given by

$$\Lambda = \tfrac{1}{2}\{-(H_{11} + H_{22}) \pm [(H_{11} + H_{22})^2 - 4(H_{11}H_{22} - H_{12}H_{21})]^{1/2}\}. \quad [40]$$

The stable condition of side-band disturbance is $\Lambda < 0$, and this condition can be written as

$$D_1 < 0. \quad [41]$$

In this study, from the definition of D_1, we find that the linear stable condition of side-band disturbance is independent of the effect of phase change. Also, we find the supercritical wave ($d_i > 0$, $E_1 > 0$) is stable with regard to side-band disturbance ($D_1 < 0$).

5. RESULTS AND DISCUSSION

The linear stability analysis yields the neutral stability curve which separates the α–Re plane into two regions; namely, the linear unstable region where small disturbances grow with time and the

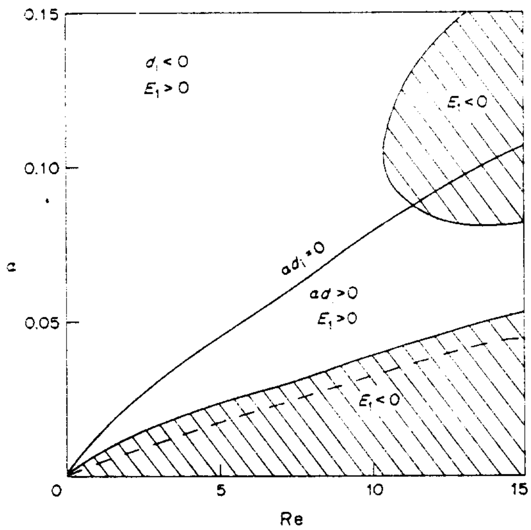

Figure 2. Stability curve of isothermal film flow: ---, side-band neutral stability curve. $W = 12{,}347$.

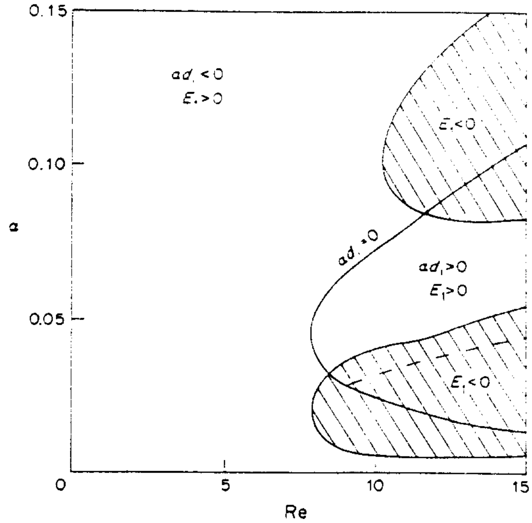

Figure 3. Stability curve of condensate film flow: ---, side-band neutral stability curve. $\xi = 0.0872$, $\mathrm{Pr} = 2.62$, $W = 12{,}347$.

linear stable region where small disturbances decay with time. For the purpose of numerical calculations, we take the temperature at the interface to be $T_s = 373$ K, and the temperature difference between the wall and interface as $T_s - T_w = \pm 47$ K. Under such temperature conditions, the values of related nondimensional parameters are $W = 12{,}347$ and $|\xi| = 0.0872$ (with phase change), $|\xi| = 0$ (without phase change). Figure 2 shows the neutral stability curve for an isothermal film flow which consists of a straight line ($\alpha = 0$) and a curve, these two lines intersect at the critical point $\alpha = 0$, $R = 0$. Figure 3 shows the neutral stability curve for condensate film flow which has a limit point corresponding to a critical Reynolds number,

$$\mathrm{Re}_{\mathrm{Cr}} = \left\{ \frac{15}{4} \left[\frac{W\xi}{3\mathrm{Pr}} \left(1 - \frac{\xi}{3} \right) \right]^{1/2} \right\}^{6/11} \quad \text{for} \quad \xi > 0. \qquad [42]$$

If the Reynolds number is smaller than the critical Reynolds number, then the disturbance of any mode is linearly stable. Figure 4 shows the linear neutral curve for an evaporating film flow which

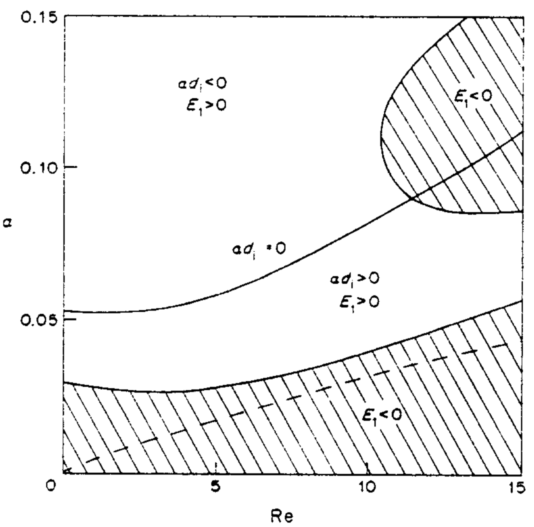

Figure 4. Stability curve of evaporate film flow: ---, side-band neutral stability curve. $\xi = -0.0872$, $\mathrm{Pr} = 2.62$, $W = 12{,}347$.

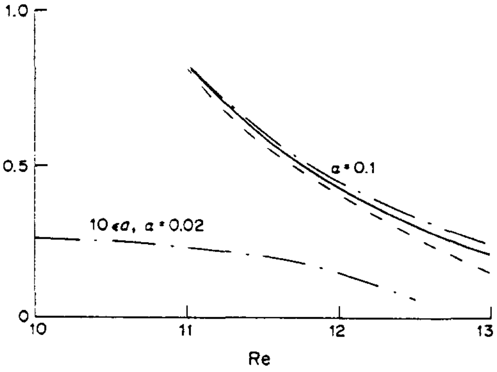

Figure 5. Threshold amplitude in the subcritical unstable region: —·— ($\xi = 0.0872$), condensation; ——, isothermal; --- ($\xi = -0.0872$), evaporation. $W = 12{,}347$, $\mathrm{Pr} = 2.62$.

has a bottom corresponding to a cut-off wavenumber. If the wavenumber is smaller than the cut-off wavenumber, then the disturbance at any Reynolds number is linearly unstable.

From [28], we find that the increase in the parameters of surface tension, W, and phase change, ξ, have a stabilizing effect on the film flow system. The increase in the Prandtl number plays a dual role: stabilizing the evaporating film flow but destabilizing the condensate film flow. These linear theory results are, in general, in agreement with the results of previous studies (Ünsal & Thomas 1978; Spindler 1982; Kocamustafaogullari 1985).

The nonlinear stability analysis is used to study whether the finite-amplitude disturbance in the linear stable region will cause instability (subcritical instability), as well as to study whether the subsequent nonlinear evolution of disturbance in the linear unstable region will develop into a new equilibrium with finite amplitude (supercritical stability) or grow to explosion. By inspection of the characteristics of [34], one finds the negative value of E_1 will make the system unstable. Such instability in the linear stable region is called subcritical instability; i.e. when the disturbance amplitude is larger than the threshold amplitude, then the amplitude will increase although the prediction by linear theory is stable. On the other hand, such instability in the linear unstable region will cause the system to reach an explosive state which could be considered as the solution of a complex pattern.

As shown in figures 2–4 by the hatched area near the neutral stability curve, it is observed that both subcritical instability ($d_i < 0$, $E_1 < 0$) and the explosive solution ($d_i > 0$, $E_1 < 0$) are possible for the isothermal, condensate and evaporating film flows. It is observed that supercritical stability ($d_i > 0$, $E_1 > 0$, unhatched area in the linear unstable region) is also possible. In such a case, filtered waves are always linearly stable with regard to side-band disturbance.

It is interesting to note that Anshus (1965) predicted subcritical instability for isothermal film flow, but he did not discover the explosive solution in the region near the lower branch of the neutral stability curve. On the other hand, Lin (1974) found that explosive solutions having the form of a solitary wave are possible, the Ünsal & Thomas (1980) indicated that subcritical instability is not possible for condensate film flow. In our view, these results of previous studies perhaps expressed some aspects of the system but did not give a complete picture. Also, Ünsal & Thomas (1980) did not accurately treat the result of second harmonic resonance because they focused on the expression of coefficient b of B but not the complete form of bA^2 in equation (29) of their paper. If we check in detail the results of their equations (29), (31) and (42), we find that the wavenumber of the second harmonic resonant will not have as simple a form as that of equation (40) in their paper.

Figure 5 displays the threshold amplitude in the subcritical unstable region with different values of ξ. It is found that decreasing the phase change parameter or increasing the Reynolds number will decrease the amplitude. Also, the threshold amplitude in the region near the lower branch of the neutral stability curve is smaller than that in the region near the upper branch of the neutral stability curve for the case of condensate film flow.

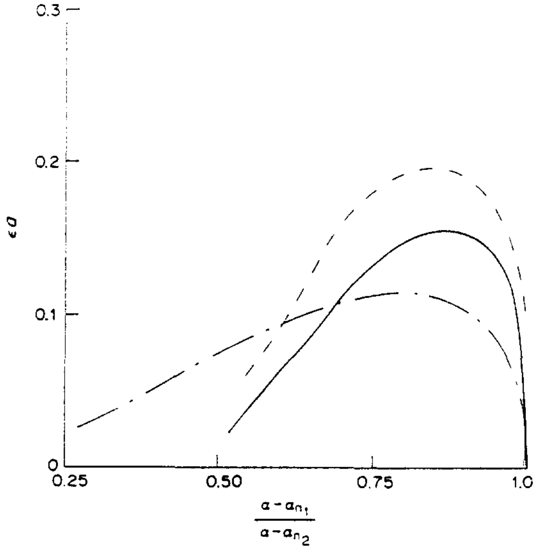

Figure 6. Amplitude of a supercritical wave: —·— (ξ = 0.0872), condensation; ——, isothermal; --- (ξ = −0.0872), evaporation. Re = 10, W = 12,347, Pr = 2.62.

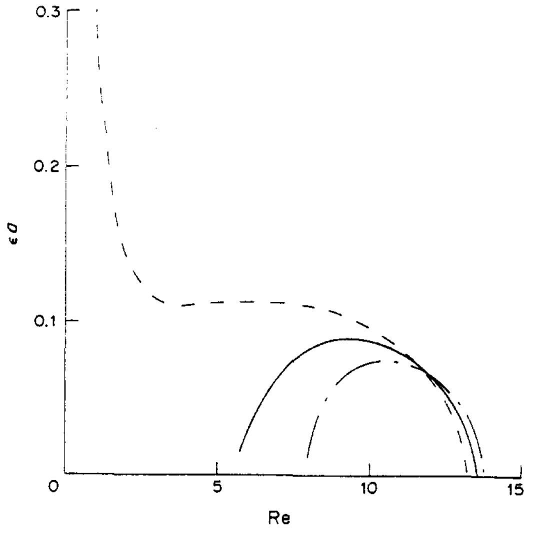

Figure 7. Amplitude of a supercritical wave: —·— (ξ = 0.0872), condensation; ——, isothermal; --- (ξ = −0.0872), evaporation. α = 0.05, W = 12,347, Pr = 2.62.

Figures 6 and 7 display the finite amplitude of a supercritical wave. The parameters α_{n_2} and α_{n_1}, in figure 6, indicate the wavenumber at the upper and lower bounds of the linear unstable region in the α–Re plane. We find that increasing the phase change parameter reduces the magnitude of the finite amplitude. As already explained by Ünsal & Thomas (1978) and Spindler (1982), the kinetic interpretation is as follows. Examining, for instance, the case of condensation (evaporation), the film thickness at the trough of a wave is smaller than at the crest. Hence, the rate of phase change is a little larger at the trough and, consequently, an excess of condensing liquid (evaporating vapor) appears compared to what happens at the crest. This leads to a smaller (larger) wave amplitude. It is also of note that the relative region of existence of a supercritical wave for condensate film flow is larger than that for isothermal and evaporate film flow.

Figure 8 shows that the increase in the magnitude of the wave speed $\epsilon^2 b$, due to nonlinear effects, is weakly affected by the phase change parameter but is strongly enhanced by increasing Reynolds number Re.

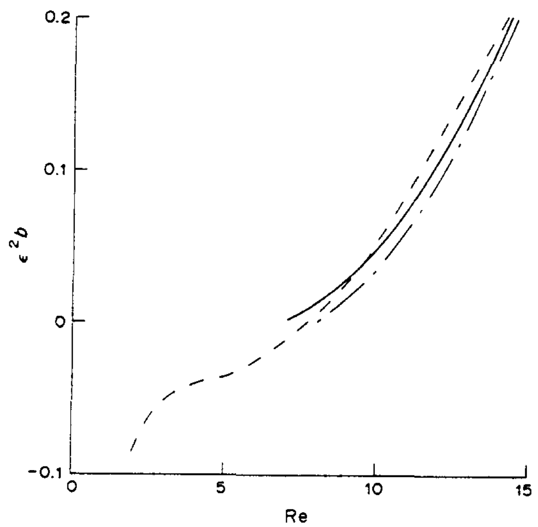

Figure 8. The increment of wave speed: —·— (ξ = 0.0872), condensation; ——, isothermal; --- (ξ = −0.0872), evaporation. α = 0.1, W = 12,347, Pr = 2.62.

6. CONCLUSIONS

The stability of film flow with phase change at an interface is investigated by the method of perturbation. In linear theory, any Reynolds number is unstable for the cases of isothermal and evaporate film flow and a finite critical Reynolds number exists in the case of condensate film flow. In nonlinear theory, it is shown that both subcritical instability and supercritical stability are possible.

For the cases of isothermal and evaporating film flow, subcritical instability is possible only in the region near the upper bound of the linear stable region in the α–Re plane. But there are two regions, one near the upper bound and the other near the lower bound of the linear stable region, where subcritical instability is possible in the case of condensate film flow. Increasing the Reynolds number or decreasing the phase change parameter will reduce the threshold amplitude in the subcritical unstable region.

The supercritical wave is possible in the region between the two explosive regions and the upper bound of the linear unstable region. It is found that an increase in the phase change parameter reduces the amplitude of the supercritical wave, while, increasing the Reynolds number increases the nonlinear wave speed. Also, the supercritical filtered waves are always linear stable with regard to side band disturbance and the mechanism of side-band disturbance, in our study, is independent of the effect of phase change.

Finally, we conclude that the effect of mass transfer at the interface will strongly modify the stability characteristics of the film flow when phase change is considered, and condensation (evaporation) is more stable (unstable) than in the case of isothermal film flow.

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